OPERATIONAL CONTROL OF PROBABILISTIC PROCESSES IN A MULTI-STAGE SPECIALIZED SYSTEM

O. Ja. Kravets (1), I. A. Aksenov (2), P. A. Rahman (3), I. V. Atlasov (4), Yu. V. Redkin (5), O. Yu. Frantsisko (6), O. Yu. Zaslavskaya (7)*

(1) Voronezh State Technical University, Voronezh; (2) Vladimir State University named after Alexander and Nikolay Stoletovs, Vladimir; (3) Ufa State Petroleum Technological University, Ufa; (4) Moscow University of Ministry of Internal Affairs of Russian Federation named by V. Ja. Kikot, Moscow; (5) Admiral Ushakov Maritime State University, Novorossiysk; (6) Kuban State Agrarian University named after I.T. Trubilin, Krasnodar; (7) Moscow City University, Moscow
Russian Federation

* Corresponding Author, e-mail: csit@bk.ru

Abstract: A process consisting of sequential operations is considered. For each operation, you can determine the currently available probability of its error-free (high-quality) execution. By the qualitative performance of the operation, we will understand the achievement of a given level of quality in quantitative proportion to the overall level. The purpose of the article is to build a quality management strategy in which a set of probabilities of quality levels not less than a certain threshold value is achieved, with minimal costs for carrying out appropriate measures. The article presents the decomposition of operations into activities; classification of activities into activities with conditionally linear or asymptotic response; quality assessment in a multi-level control system with response functions of various types; optimization problems and their algorithmic solutions.

Key words: sequential operations, high-quality execution, quality management strategy, threshold value, classification of activities.

1. INTRODUCTION

In accordance with the task of improving quality in a multi-level control system, it is necessary not only to decide what is more effective for improving quality at each stage of control and the chain of execution of decisions made, which makes it possible to determine not only the focus of financing and its ways, but also to calculate the achieved level of quality improvement [1]. In some cases, it may be optimal, in others, despite many-fold financial investments, it may only approach it.

Let's consider the "quality pipeline" from the point of view of the most rational ways to improve quality in a multi-level control system. Modern control complexes of complex systems, to which, without a doubt, multilevel control systems belong, provide an approximation to the limit of the quality level in the overall control system. In the conditions of the "quality pipeline" functioning in control systems, it is really necessary to deal with a certain proportion of control decisions, according to which all planned activities are carried out in full and with the highest quality. In fact, operating with the percentage of tasks completed, we mean the current probability of this completion, and - importantly – with the percentage of effectiveness of management decisions made.

The current probability of the effectiveness of the implementation of the decisions taken on the "quality pipeline" in the vast majority of cases is less than 1, which is due to a number of reasons.

We will consider a multi-level control system as a sequence of operations.

The purpose of the article is to build a quality management strategy in which a set of probabilities of quality levels not less than a certain threshold value is achieved, with minimal costs for carrying out appropriate measures.

2. PREVIOUS RESEARCH

A process consisting of sequential operations is considered. For each operation, you can determine the currently available probability of its error-free (high-quality) execution. By the qualitative performance of the operation, we will understand the achievement of a given level of quality in quantitative proportion to the overall level. The goal of quality management is to achieve control probabilities that are not less than a certain threshold value of the quality level, with minimal costs for carrying out appropriate measures.

Similar processes and their analysis were considered in [2] (deterministic parameters), [3] (local optimization), [4] (multi-criteria optimization for solving a specific problem), [5] (workflow optimization).

The article presents the decomposition of operations into activities; classification of activities into activities with conditionally linear or asymptotic response; quality assessment in a multi-level control system with response functions of various types; optimization problems and their algorithmic solutions.

3. OPERATIONS AND ACTIVITIES

Considering the operations to be independent in aggregate and using the probability multiplication theorem [6], we obtain that the probability of a qualitative completion of the entire sequence of operations is a production

$$p^0 = \prod_{i=1}^{n} p_i^0 \cdot \quad (1)$$

The probability of high-quality performance of each operation can be increased by carrying out activities. However, the ideal value of such a criterion, equal to one, can be achieved only if all probabilities are equal to 1, which is usually not practically achievable. Therefore, it is proposed to choose a certain threshold value of the probability of a qualitative completion of the control process, close to one, and by controlling the probabilities p_{before} of individual processes to achieve this threshold value, that is, to transfer the target function to the category of constraints. The events can be divided into two groups:

• events, the "unit" of which increases the corresponding probability by the same value, independent of the initial value, until the theoretical limit (unit) is reached - with a conditionally linear response;

• events, the "unit" of which increases the corresponding probability by an amount the smaller the closer this probability is to the unit - with an asymptotic response.

Each event requires a certain amount of resources to conduct it. Thus, the probability of high-quality performance of the operation is a function of the amount of invested funds.

Figures 1 and 2 reflect the dependence of the expected relative cost of measures with a conditionally linear and asymptotic response, respectively, on the required probability of p_q in terms of a fixed increment of probability. Carrying out measures with a conditionally linear response to improve the quality of control stages gives predictable results on each individual segment of the p_q scale.

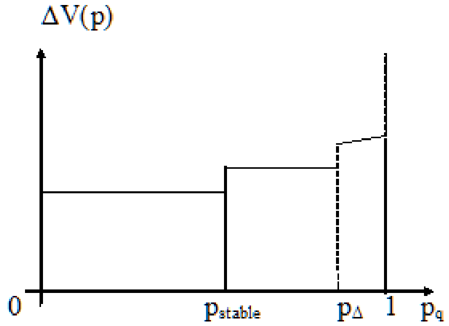

Figure 1. Typical dependence of the expected relative cost of measures with a conditionally linear response on the required probability of a swing in terms of a fixed increment of probability p_q

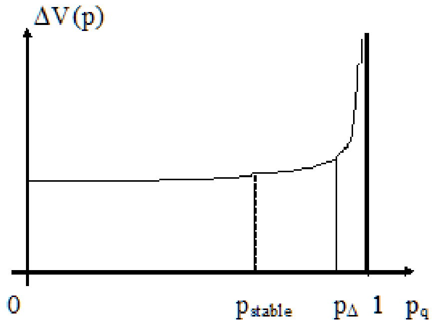

Figure 2. The typical dependence of the expected relative cost of measures with an asymptotic response on the required probability of p_q in terms of a fixed increment of probability

The almost unlimited growth of the necessary increment of costs to increase the probability in Figure 2 indicates the fundamental unattainability of absolutely high-quality multi-level control for events with an asymptotic response. The necessity of these activities creates the need to take into account their impact on the quality of control results. Solving the problem can also be useful in purely technical applications [7, 8].

4. QUALITY ASSESSMENT IN A MULTI-LEVEL CONTROL SYSTEM WITH CONTINUOUS RESPONSE FUNCTIONS

We investigate the problem with various types of dependence of the probability of high-quality execution of the operation on the amount of invested funds. Let's consider events with a conditionally linear response. The dependence of the probability p_i of the qualitative performance of the i-th operation on the amount of invested funds can be conditionally depicted graphically (Figures 3, 4).

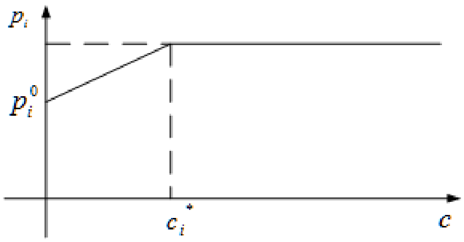

Figure 3. Dependence of the probability on the invested funds with a conditionally linear response

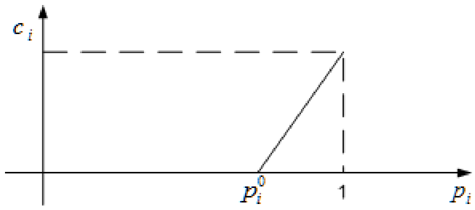

Figure 4. The dependence of the cost of the event on the desired probability for a conditionally linear response

The dependence of probability on c_i is represented by the equation

$$p_i(c_i) = p_i^0 + \frac{1 - p_i^0}{c_i^*} \cdot c_i; \; c_i \in [0, c_i^*]. \tag{2}$$

Then the inverse function has the form

$$c_i(p_i) = \frac{c_i^*}{1 - p_i^0} \cdot (p_i - p_i^0); \; p_i \in [p_i^0, 1]. \tag{3}$$

It is necessary that the probability of high-quality performance of all operations is not less than the limit value p_{limit}. We will consider the operations to be independent in aggregate, then

$$\prod_{i=1}^{n} p_i \geq p_{before}. \tag{4}$$

Adding up the cost of activities for all operations, we get the total cost

$$c(p) = \sum_{i=1}^{n} c_i(p_i) = \sum_{i=1}^{n} \frac{c_i^*}{1 - p_i^0}(p_i - p_i^0). \tag{5}$$

We come to the following task: minimize (5) when performing (4) and

$$p_i^0 \leq p_i \leq 1 \quad i = 1, \ldots, n. \tag{6}$$

This problem is a problem of minimizing a linear function with nonlinear constraints. To solve the problem, we will compose a Lagrange function. To do this, we will rewrite the restrictions in the form

$$p_i \leq 1;\ i = 1,...,n. \tag{7}$$

The left constraint inequality (6) can be discarded, because it is automatically executed when inequality (4) is executed. Taking into account the inequalities (4), (7), the Lagrange function will have the form

$$L(p_1,...,p_n;\lambda_1,...,\lambda_{n+1}) = \\ = \sum_{i=1}^{n} \frac{c_i^*}{1-p_i^0}(p_i - p_i^0) + \sum_{i=1}^{n} \lambda_i (1-p_i) + \lambda_{n+1}(p_{before} - \prod_{i=1}^{n} p_i). \tag{8}$$

The necessary conditions of the extremum are the Kuhn-Tucker conditions:

$$\frac{c_i^*}{1-p_i^0} - \lambda_i - \lambda_{n+1} \prod_{\substack{j=1 \\ j \neq i}}^{n} p_j = 0;\ i = 1,...,n, \tag{9}$$

$$\prod_{i=1}^{n} p_i \geq p_{before} \tag{10}$$

$$\lambda_i (1-p_i) = 0;\ i = 1,...,n \tag{11}$$

$$\lambda_{n+1}(p_{before} - \prod_{i=1}^{n} p_i) = 0 \tag{12}$$

$$\lambda_i \geq 0;\ i = 1,...,n+1. \tag{13}$$

Since the objective function is linear, it reaches the smallest value at the boundary of the domain defined by conditions (4), (6).

With $\lambda_{n+1} = 0$ (9) we obtain that $\lambda_i \neq 0$, $i = 1,...,n$. Then the solution of the system will be $p_i = 1$, $i = 1,...,n$.

When $\lambda_{n+1} \neq 0$ from equation (12) we have $p_{before} = \prod_{i=1}^{n} p_i$, i.e. condition (4) is fulfilled as equality. Equating to zero various combinations of Lagrange multipliers λ_i, $i = 1,...,n$, we obtain a finite number (no more than $1 + \sum_{k=1}^{n} C_n^k$) solutions of the system (9)-(13).

In other cases, the system is incompatible. Calculating the values of the objective function at the obtained points, we find the smallest. These will be the desired values of the probabilities of high-quality operations, obtained at minimal cost.

Thus, under the condition of linear dependence of probabilities on the invested funds, the problem can be solved analytically without the numerical methods.

Consider the case of an asymptotic response. Suppose that the dependence of the probability of a qualitative completion of the operation on the cost of the event is described by an exponential function (without limiting generality, we will assume the exponent as the basis, Figure 5), and is described by a dependence of the form

$$p(c) = 1 - (1 - p_0)e^{-\alpha c} \tag{14}$$

where $\alpha > 0$ is the coefficient characterizing the curvature of the function.

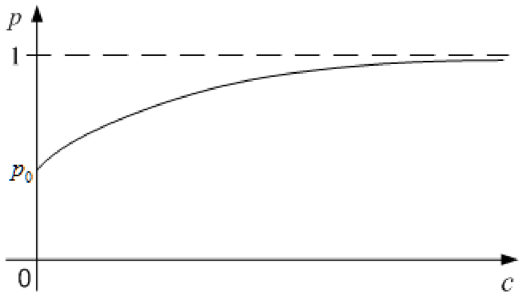

Figure 5. Dependence of the probability on the invested resources with an asymptotic response

Then the inverse function will have the form

$$c(p) = -\frac{1}{\alpha}\ln\frac{1-p}{1-p_0} \text{ or } c(p) = \ln\left(\frac{1-p_0}{1-p}\right)^{\frac{1}{\alpha}} \tag{15}$$

The cost of performing all operations will be equal to

$$C(p) = \sum_{i=1}^{n} \ln\left(\frac{1-p_i^0}{1-p_i}\right)^{\frac{1}{\alpha_i}} = \ln\prod_{i=1}^{n}\left(\frac{1-p_i^0}{1-p_i}\right)^{\frac{1}{\alpha_i}} \tag{16}$$

Since it is a strictly increasing function over the entire domain of definition, instead of the function (18) we will consider

$$C'(p) = \prod_{i=1}^{n}\left(\frac{1-p_i^0}{1-p_i}\right)^{\frac{1}{\alpha_i}} \tag{17}$$

Thus, we come to the optimization problem:

$$\prod_{i=1}^{n}\left(\frac{1-p_i^0}{1-p_i}\right)^{\frac{1}{\alpha_i}} \to \min \tag{18}$$

$$\prod_{i=1}^{n} p_i \geq p_{before} \tag{19}$$

$$p_i^0 \leq p_i \leq 1 \qquad i = 1,\ldots,n \tag{20}$$

To solve the problem, as in the case of a linear response, we will use the Lagrange multiplier method. The system is converted to the form

$$\begin{cases} \left(\alpha_1(1-p_1)\cdot\prod\limits_{\substack{j=1\\ j\neq 1}}^{n} p_j\right)^{-1} = \left(\alpha_2(1-p_2)\cdot\prod\limits_{\substack{j=1\\ j\neq 2}}^{n} p_j\right)^{-1} = \ldots = \left(\alpha_n(1-p_n)\cdot\prod\limits_{\substack{j=1\\ j\neq n}}^{n} p_j\right)^{-1} \\ \prod\limits_{j=1}^{n} p_j = p_{before} \end{cases} \quad (21)$$

5. CONCLUSION

1. The proposed concept of probabilistic quality management in a multi-level control system is based on the assumption that there is some stable level (probability) of obtaining correct control results and differs in ways of influencing this probability.

2. The formulation of the optimization model of the task of operational control in the system of multilevel control of systems is based on the concept of probabilistic control. A distinctive feature of the formulation is its multilevel nature and the possibility of taking into account the effectiveness of the impact on the control quality.

3. The presentation of the control system as a conveyor system provides the possibility of separate influence on the components of the control process, depending on the priority for each given time period.

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Information about the authors:

Oleg Jakovlevich Kravets – Dr. Sci (IT), professor of Voronezh State Technical University, areas of scientific research - system analysis, optimization, simulation of complex objects

Ilia Antonovich Aksenov – PhD in economics, associate professor of Vladimir State University named after Alexander and Nikolay Stoletovs, areas of scientific research - economic information systems

Pavel Azizurovich Rahman – PhD in technical sciences, associate professor of Ufa State Petroleum Technological University, areas of scientific research - information technologies, computer networks, reliability modeling and engineering

Igor Victorovich Atlasov – Dr. Sci (Phys.-Math.), professor of Moscow University of Ministry of Internal Affairs of Russian Federation named by V. Ja. Kikot, areas of scientific research - system analysis, optimization, simulation of complex objects

Yuriy Viktorovich Redkin – PhD, associate professor of Admiral Ushakov Maritime State University, Novorossiysk, areas of scientific research - wireless networks, digital signal processing, control and data processing systems

Olga Yurievna Frantsisko - PhD (Econ.), associate professor of Kuban State Agrarian University named after I.T. Trubilin, Krasnodar, areas of scientific research - economic and mathematical modeling, scenario approach, processing industries of agricultural enterprises, institutional transformations of the agricultural sector of the economy, information technology, management of the agricultural sector

Olga Yurievna Zaslavskaya – Dr. Sci (Pedagogy), professor of Moscow City University, Institute of Digital Education, Department of Education Informatization, areas of scientific research – informatization of education, theory and methodology of training and education, managerial competence of a teacher

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